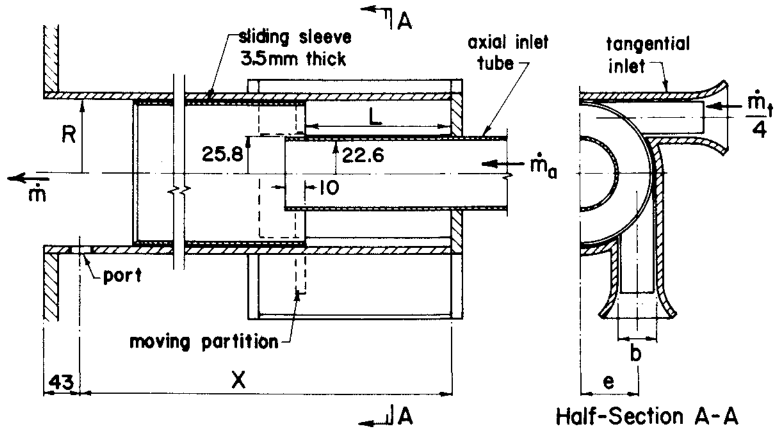

Fig. 1 The axial-tangential swirl generator. Dimensions: $b=25.6$ mm, $e=38.0$ mm, $L=$variable, $R=50.8$ mm, $X=529$ mm (other dimensions in mm).

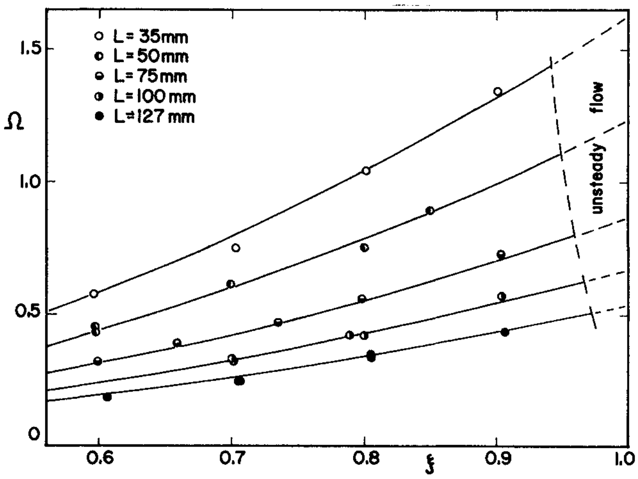

Fig. 2 Dimensionless angular momentum flux Ω as function of *L* and ξ.

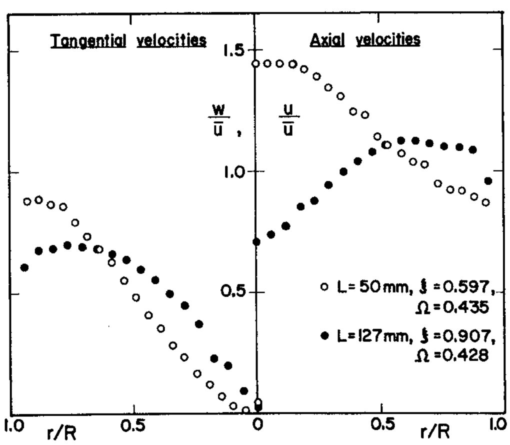

Fig. 3 Profiles of axial and tangential velocities at approximately constant Ω and various inlet lengths *L*.

ing (T/IR) gives the outlet Ω as

$$\Omega=\pi\xi^2 R\left[\frac{e}{nbLC_c}-\frac{\pi f_\phi R^2 X}{4n^2b^2L^2C_c^2}\right] \tag{10}$$

Least squares fitting gave $C_c=0.82$ and $f_\phi=0.022$, the latter being about twice that for simple smooth pipe flow at the same *Re*. This equation is plotted in Fig. 2, and fits all experimental points within 7%.

Acknowledgments

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Vibration of a Large Space Beam Under Gravity Effect

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Introduction

MOST spacecraft launched to date have undergone structural ground tests to verify the mathematical models of the structures.[1-2] The effect of gravity on these traditional space structures was considered only from a static point of view, or completely disregarded. This is because of their stiffness and compactness, and also because they were

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designed to survive the launch loads. However, the structures to be used for future space applications will be very large, such as a manned space station or a wrap-rib antenna.[3] These space structures may have dimensions of the order of 30-200 m. Such space structures will have a low mass density and will be highly flexible. Their dynamic behavior in ground tests (1-g field) might be quite different from that in orbit (0-g field). It is this kind of problem that is addressed in this investigation.

Since many flexible space structures can be modeled as beams, the generic structural element chosen for this study is a space beam with a large slenderness ratio. The nonlinear vibrations of beams have been studied by many investigators.[4-6] However, these studies are focused on the vibrations of underformed beams. The present work investigates the vibration behavior of a beam deformed by its own weight. The differential equations for both the static and dynamic responses of a large, simply supported beam subjected to its own weight are derived and solved analytically. The results enable one to verify the dynamic characteristics of a beam-type structure in orbit by using the experimental measurements of ground tests. It should also be pointed out that a flexible-beam flight experiment is being planned by NASA as part of the Control of Flexible Structure (COFS) program.

Static Analysis

The governing differential equations of a large, simply supported beam subjected to its own weight (Fig. 1) can be written as

$$\frac{dN}{dx}=0 \tag{1}$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2W}{dx^2}\right)-N\frac{d^2W}{dx^2}=\rho gA \tag{2}$$

where N is the axial stretching forces, E is Young's modulus, I is the moment of inertia, W is the static deformation, ρ is the beam density, g is the acceleration of gravity, and A is the beam cross section.

From the governing equations and its corresponding boundary conditions, the static deformation W can be written in terms of the axial stretching force as

$$W(x)=\frac{\rho gA}{\lambda^2N}\left[\frac{\cosh(\lambda L/2)(1-2x/L)}{\cosh(\lambda L/2)}-1\right]+\frac{\rho gAL^2}{2N}\left[\frac{x}{L}\left(1-\frac{x}{L}\right)\right] \tag{3}$$

where L is the beam length and $\lambda=(N/EI)^{1/2}$.

The stretching force N can be derived from the relation

$$\int_0^L\frac{du}{dx}dx=\int_0^L\left[\frac{N}{EA}-\frac{1}{2}\left(\frac{dW}{dx}\right)^2\right]dx=0 \tag{4}$$

Substituting Eq. (3) into Eq. (4), one can determine the stretching force N by the formulation

$$\frac{16N^3\mu^3}{(\rho gA)^2EAL^2}=\frac{2}{3}\mu^3-4\mu+5\tanh\mu-\frac{\mu}{\cosh^2\mu} \tag{5}$$

where $\mu=\lambda L/2$.

Free Vibration

The deflection function of the dynamic response can be written

$$W_d(x,t)=W(x)+W_b(x,t) \tag{6}$$

where $W(x)$ is the static deformation and $W_b(x,t)$ is the vibration response. For simplicity, the static deformation and vibration response can be expressed by

$$W(x)=\sum_{i=1,3,\ldots}W_i\sin\left(\frac{i\pi x}{L}\right) \tag{7}$$

$$W_b(x,t)=a(t)\sin(n\pi x/L) \tag{8}$$

where W_i, $i=1,3,5,\ldots$, is a series coefficient that can be determined from Eq. (3) and n is the vibration mode number.

The governing equation of a free vibration can be expressed by

$$EI\frac{\partial^4W_d}{\partial x^4}-N_d\frac{\partial^2W_d}{\partial x^2}=\rho A\frac{\partial^2W_d}{\partial t^2}+\rho gA \tag{9}$$

where the stretching force N_d can be written

$$N_d=\frac{EA}{2L}\int_0^L\left(\frac{\partial W_d}{\partial x}\right)^2dx \tag{10}$$

Combining the equations described and applying the orthogonality property, one can summarize the governing equations for the free vibration of a beam subject to its own weight as follows:

$$\ddot{a}(t)+g_1a(t)+g_2a^2(t)+g_3a^3(t)=0 \tag{11}$$

where

$$g_1=\left(\frac{n\pi}{L}\right)^4\frac{EI}{\rho A}\left(1+\frac{NL^2}{n^2\pi^2EI}+\frac{AW_i^2}{2I}\right)\quad \text{for } n=i=1,3,5,\ldots \tag{12a}$$

$$g_1=\left(\frac{n\pi}{L}\right)^4\frac{EI}{\rho A}\left(1+\frac{NL^2}{n^2\pi^2EI}\right)\quad \text{for } n=2,4,6,\ldots \tag{12b}$$

$$g_2=\left(\frac{n\pi}{L}\right)^4\frac{EI}{\rho A}\frac{3AW_i}{4I}\quad \text{for } n=i=1,3,5,\ldots \tag{13a}$$

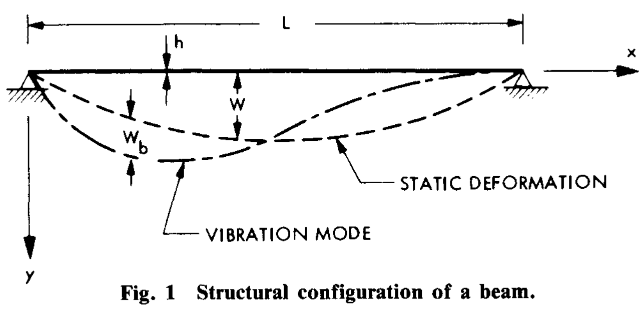

Fig. 1 Structural configuration of a beam.

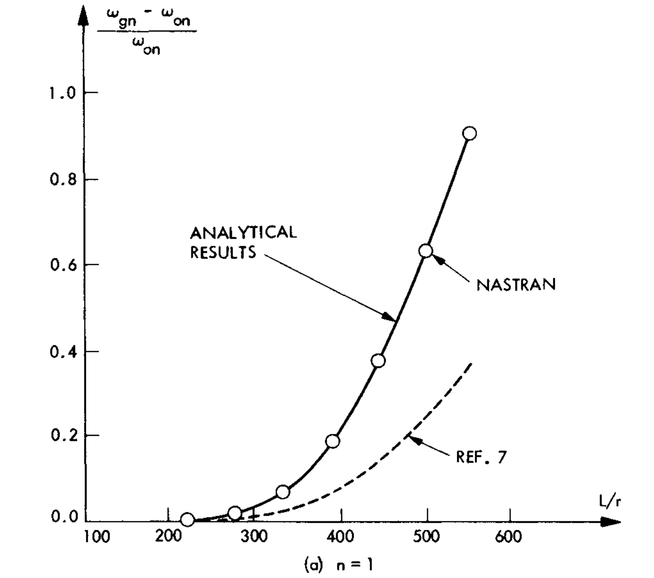

Fig. 2 Comparison of frequency variations.

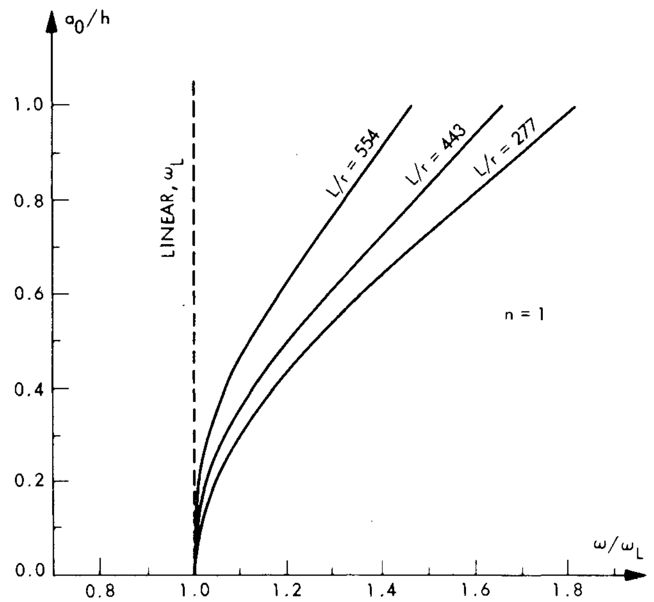

Fig. 3 Amplitude-vs-frequency plot.

$$g_2 = 0 \qquad \text{for } n = 2,4,6,... \tag{13b}$$

$$g_3 = \left(\frac{n\pi}{L}\right)^4 \frac{EI}{\rho A} \frac{A}{4I} \qquad \text{for all } n \tag{14}$$

First, let us assume that the excitation amplitude of $a(t)$ is much smaller than the maximum static deformation $W(x)$. Then the governing equation can be linearized by neglecting the higher-order terms of $a(t)$. Assuming simple harmonic excitation, $\ddot{a}(t) = -\omega^2 a(t)$, one can express the frequency equations by

$$\frac{\omega_{gn}^2}{\omega_{0n}^2} = 1 + \frac{NL^2}{n^2\pi^2 EI} + \frac{AW_i^2}{2I} \qquad \text{for } n = i = 1,3,5,... \tag{15a}$$

$$\frac{\omega_{gn}^2}{\omega_{0n}^2} = 1 + \frac{NL^2}{n^2\pi^2 EI} \qquad \text{for } n = 2,4,6,... \tag{15b}$$

where ω_{gn} is the natural frequency of the nth mode due to the effect of gravity and ω_{0n} is the natural frequency of the nth mode in a 0-g environment.

Equations (15a) and (15b) indicate that the natural frequency of the symmetric mode ($n = 1,3,5,...$) depends upon not only the axial stretching force but also the static deformation due to the beam's own weight. However, Eq. (15b) shows that the natural frequency of the asymmetric mode ($n = 2,4,6,...$) is not affected by the static deformation. For simplicity, an aluminum beam with a cross section of 1 in. × 1 in. is applied. The natural frequencies calculated from Eqs. (15a) and (15b) are shown in Fig. 2 as a function of the slenderness ratio. The results obtained from NASTRAN are also plotted in Fig. 2 for comparison, and the results from the linearized approach agree well with those from NASTRAN runs. In addition, Fig. 2 also shows the results predicted by using the frequency equation derived in Ref. 7. The comparison indicates that the results from Ref. 7 are not satisfactory. This is because the nonlinear coupling due to the static deformation is not considered in Ref. 7.

The nonlinear free vibration of Eq. (11) can be analyzed by introducing a velocity variable $v = da/dt$ and integrating Eq. (11) in a straightforward manner. The vibration period can be calculated by integrating along a trajectory on the v-a phase plane. The trajectories on the phase plane indicate that the maximum vibration amplitudes of the symmetric modes ($n = 1,3,5,...$) in different phases are not identical. The ratio of maximum negative amplitude to maximum positive amplitude depends upon both the slenderness ratio and the magnitude of the excitation amplitude. However, the trajectories of the asymmetric modes ($n = 2,4,6,...$) are symmetric with respect to both the a and v axes. The amplitude-vs-frequency plots for the nonlinear vibration of the first mode is shown in Fig. 3. The results indicate that this is a hardening system, since the frequency increases with vibration amplitude.

Conclusions

The structural characteristics of a large space beam subjected to the effect of gravity is investigated. In order to take the gravity effect into consideration, geometric nonlinearity is included in the analysis. The governing differential equation of dynamic analysis is derived by using Galerkin's method. The results of this work describe the details of both the static and the dynamic behavior of a large space beam in a 1-g environment. The results also enable one to verify the dynamic characteristics of a space beam in orbit by using the experimental results of ground tests. The numerical results based on the NASTRAN code agree well with the analytical solution from both static and linearized dynamic approaches. It is noted that the applications of this code are limited to problems of small vibration. A small vibration is defined by its relative vibration amplitude in response to the static deformation due to its weight.

Acknowledgments

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